March 29, 1995 WOCOMAL VARSITY MEET

ROUND I: Elementary number theory

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. What is the largest prime factor of 1995?

2. 49_B represents a two digit number in base B. If the number 94_B is double 49_B, find B.

3. The same structure which holds when representing numbers in different bases also works for negative integer bases. Find the base -3 representation for 47_{10}. (as with base 3, base -3 only uses the digits 0,1, and 2.)

ANSWERS

(1 pt) 1.____________________

(2 pts) 2.____________________

(3 pts) 3.____________________

Auburn, Bartlett, Mass. Academy

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ROUND II: Algebra I- open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Solve for *k*: $\left(3^4\right)\left(2^6\right)=4\left(6^k\right)$

2. There are 20 questions on a math test. The student earns three points for each correct answer and loses one point for each wrong answer or unanswered question. A student with a score of 32 had how many questions correct?

3. Working together, machines A and B can do a job in 9 hours. Machines B and C can do the same job in 12 hours working together. When A, B, and C all work together, the job is completed in 6 hours. How long would it take machine B, working alone, to do the job?

ANSWERS

(1 pt) 1. $k=$ ____________

(2 pts) 2. ____________

(3 pts) 3. ____________ hours

St. John's, West Boylston, Westboro

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ROUND III: Theory of polynomial equations and functions, complex numbers

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. Evaluate: $i^{97}+i^{98}+i^{99}+i^{100}$

2. Given that $f(x)=x^2+3x+5$ and that p and q are such that $f(p+q)=f(p)+f(q)$, find the value of the product pq.

3. If a and b are real numbers and $\frac{1}{a+bi} = \frac{1}{2} - \frac{1}{3}i$, evaluate $\frac{a}{b}$.

ANSWERS

(1 pt) 1.____________________

(2 pts) 2.____________________

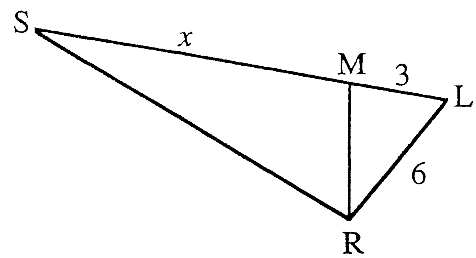
(3 pts) 3.____________________

Auburn, Bartlett, St. John's

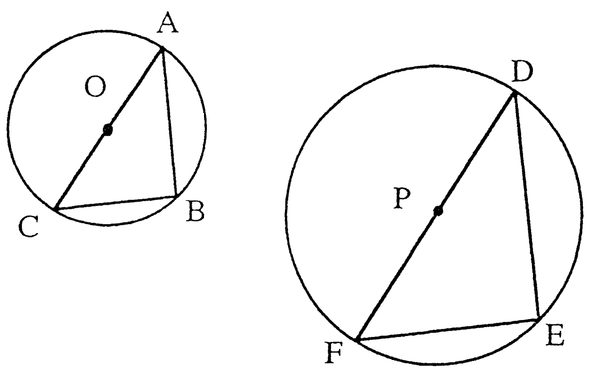
ROUND IV: Similarity and Pythagorean relationships

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO FOUR PLACES AFTER THE DECIMAL POINT

1. If $\overline{SL} = \overline{SR}$ and $\overline{RM} = \overline{RL}$, find x.



2. Similar triangles ABC and DEF are inscribed in circles O and P. If $\overline{OA}=2$, $\overline{AB}=3$, and $\overline{ED}=5$, find $\overline{EF}$



3. The medians of a right triangle that are drawn from the vertices of the acute angles have lengths $2\sqrt{11}$ and 9. Find the length of the hypotenuse.

ANSWERS

(1 pt) 1.____________________

(2 pts) 2.____________________

(3 pts) 3.____________________

Shrewsbury, Tahanto, Tantasqua, Worcester Academy

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ROUND V: Trigonometry- open

ALL ANSWERS MUST BE EXPRESSED IN THE FORM SPECIFIED IN THE PROBLEM

1. Find the exact value of $\cos x$ if

$$3^{\sec x} = 9^{\cos x} \quad \text{and} \quad 0 \le x \le \frac{\pi}{2}.$$

2. A rhombus of perimeter 40cm has a 70° angle. Find the length of the longer diagonal to the nearest tenth.

3. If A is an acute angle less than 45°, and $\sin 2A = y$, express $\sin A - \cos A$ in terms of y.

ANSWERS

(1 pt) 1.____________________

(2 pts) 2.________________ cm.

(3 pts) 3.____________________

Auburn, Bancroft, Notre Dame

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TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM, NO APPROXIMATIONS, UNLESS OTHERWISE DIRECTED IN THE PROBLEM

2 points each

1. The sport of moshball awards 7 points for a slamdunk and 9 points for a splatter. What is the smallest value of N such that all scores N and greater can be achieved in this sport?

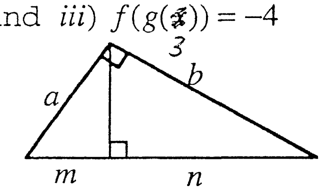
2. Find all real values of x for which $5x+3\sqrt{x}=2$

3. Find all values of k which satisfy

   $i)\ f(x)=x^2+k$

   $ii)\ g(x)=2x+k$

   and $iii)\ f(g(\overset{3}{\cancel{x}}))=-4$

4. 

   If $\dfrac{a}{b}=\dfrac{1}{3}$, find $\dfrac{m}{n}$ as a reduced fraction.

5. Solve for x where $0\le x\le 2\pi$: $2\sin^2 x + 3\sin x \ge 2$.
   Express your answers in terms of integer or fractional multiples of π.

6. Find all the ordered pairs (x,y) for which $\sqrt{x}+\sqrt{y}=17$ and $x-y=85$.

7. Find x as a fraction or mixed number, not a decimal, if

$$\frac{7\bullet 7^{\frac{1}{7}}}{7^7}=\left(\frac{1}{7}\right)^x$$

8. The operation $\oplus$ is defined as $a\oplus b=a^2+3b$. Find all pairs of positive integers (a,b) such that

$$a\oplus b=37.$$

9. The quotient of two consecutive positive even integers can be written as K+.01 where k is an integer. Find the larger of those two consecutive positive integers.

Bartlett, Bromfield, Burncoat, Doherty, Hudson, Mass. Academy, South, Worcester Academy

March 29, 1995

I

1. 19

2. 14

3. 12202 or -2111

II

1. 4

2 13

3 36

III

1. 0 or "zero"

2. $\frac{5}{2}$ or other forms

$+\frac{3}{2}$ or $+1.5$

IV

1. 9

2. $\frac{5\sqrt{7}}{3}$ or 4.4096

3 10

V

1. 1 $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{}}$ or $\sqrt{\frac{1}{2}}$ or $\sqrt{.5}$

2 16.4 cm

3 $-\sqrt{1-y}$ or $-(1-y)^{\frac{1}{2}}$

1. 48

2. $\frac{4}{25}$ or 0.16

3. -5 and -8

4. $\frac{1}{9}$

5. $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

no degrees, no decimals

6. (121, 36)

7. $\frac{41}{7}$ or $5\frac{6}{7}$

no decimals

8. 4 pairs

(1,12), (2,11), (4,7), (5,4)

9. 202